30. E. A. Polyakova and K. S. Shifrin, "Microstructure and transmittance of rains," Tr. Gl. Geofiz. Obs., No. 42 (1953).
31. P. V. D'yachenko, "Experience in using the methods of mathematical statistics to the study of the microstructure of mists and clouds," Tr. Gl. Geofiz. Obs., No. 101 (1959).
32. A. Kh. Khrgian and I. P. Mazin, "On drop size distribution in clouds," Tr. Tsentr. Aerol. Obs., No. 17, 56-61 (1952).

REFLECTIVITY OF A LAYER OF POLYDISPERSED WATER DROPS

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Spectral and integral reflectivities of a semi-infinite layer of water droplets are calculated. Particle size distribution is assumed to be of Γ-form. The results of numerical calculations for integral reflectivity are presented by a simple formula.

The calculations are based on the assumption of the "softness" of the particles $|n-i\varkappa| \approx 1$. Hulst's formula [1, 2] is valid for determining the dimensionless attentuation coefficient of such particles

$$K = 2 - \frac{4}{\rho'}\sin\rho' + \frac{4}{\rho'^2}(1-\cos\rho'), \tag{1}$$

where $\rho' = 2\rho(n-1)$, $\rho = 2\pi r/\lambda$. We will use a modification of this formula for the dimensionless absorption coefficient

$$K_a = K_{a\infty}\left\{1 - 2\left[\frac{1}{\omega^2} - e^{-\omega}\left(\frac{1}{\omega} + \frac{1}{\omega^2}\right)\right]\right\}, \tag{2}$$

where $K_{a\infty}$ is the dimensionless absorption coefficient of particles with an infinite radius:

$$\omega = 4a(n)\varkappa\rho/K_{a\infty}. \tag{3}$$

The appearance of function a(n) is shown in [3], and the following approximate expression was derived in [4]

$$a(n) = n^2\left[1 - \left(\frac{n^2-1}{n^2}\right)^{3/2}\right]. \tag{4}$$

Table 1 shows values of $K_{a\infty}$ calculated in accordance with the electromagnetic theory at $\varkappa\rho \gg 1$, $\varkappa \ll n-1$. Table 2 shows the results of calculations with Eq. (2) compared to the results of a rigorous solution. Equation (1) for K_a leads to a substantial divergence from the correct values compared to Eqs. (2) and (3).

To solve the transfer equation, apart from K and K_a we need to know the scattering function. We will limit ourselves to an approximate solution with one functional parameter, for which we will use the mean cosine of the scattering angle for minimum scattering. The following design formula was derived from an analysis of data obtained on an electronic digital computer in accordance with the Mie theory

$$\bar{\mu} = \bar{\mu}_\infty[1 - e^{-1.2\rho}(1 + 1.2\rho)], \tag{5}$$

and is valid for $n \approx 1.3$ and $\varkappa \ll 1$. The following approximate expression was derived for the

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mean cosine of optically coarse ($\rho \gg 1$), nearly transparent ($4\varkappa\rho \ll 1$) particles

$$\bar{\mu}_{\infty} = 1 - \frac{n-1}{2.25}, \tag{6}$$

which is sufficiently accurate with $1 \leqslant n \leqslant 1.7$.

We will assume that the water drops in the layer are polydispersed. Actual calculations were performed for a radius distribution density in the form of a first-order gamma distribution:

$$f(r) = Ar\,e^{-2r/\bar{r}}. \tag{7}$$

The radiative transfer equation includes the linear attenuation coefficient k, linear absorption coefficient α, and mean cosine of scattering angle for minimum scattering. These quantities have the form

$$k = \frac{CS}{4}\langle K \rangle, \quad \alpha = \frac{CS}{4}\langle K_a \rangle, \tag{8}$$

$$\langle \bar{\mu} \rangle = 4\pi N \int_0^{\infty} \bar{\mu}(r) K_s f(r)\, dr / S \langle K_s \rangle, \tag{9}$$

where $S = 4\pi N \int_0^{\infty} r^2 f(r)\, dr$ — the specific surface of the particle; N is the number of particles per unit mass of the particles

$$\langle K \rangle = 4\pi N \int_0^{\infty} r^2 K f(r)\, dr / S; \tag{10}$$

$K_s = K - K_a$. The expressions for $\langle K_a \rangle$ and $\langle K_s \rangle$ are similar to Eq. (10). Equation (1) was averaged for the gamma distribution of a whole series in [5]. It was found from [5] that

$$\langle K \rangle = 2\,\frac{5\rho_1^2 + \frac{10}{3}\rho_1^4 + \rho_1^6}{(1+\rho_1^3)^3}, \tag{11}$$

TABLE 1. Value of Coefficient $K_{a\infty}$

n	$K_{a\infty}$	n	$K_{a\infty}$
1.2	0.98	3.0	0.78
1.3	0.975	4.0	0.68
1.5	0.97	5.0	0.61
2.0	0.90	10.0	0.39

TABLE 2. Comparison of Results of Calculation of K_a by Eq. (2) with the Results of a Rigorous Solution

ρ	$n=1.2$; $\varkappa=0.1$		$n=1.3$; $\varkappa=0.00062$		ρ	$n=1.2$; $\varkappa=0.1$		$n=1.3$; $\varkappa=0.00062$	
	$K_a(2)$	K_a	$K_a(2)$	K_a		$K_a(2)$	K_a	$K_a(2)$	K_a
0.25	0.076	0.062	—	—	2.0	0.456	0.516	0.00422	0.00448
0.5	0.146	0.127	—	—	5.0	0.751	0.930	0.0105	0.0128
1.0	0.268	0.265	0.00211	0.00177	10.0	0.900	1.130	0.0211	0.0272

where $\rho_1 = (n-1)\bar{\rho}$; $\bar{\rho} = 2\pi\bar{r}/\lambda$.   For $\langle K_a \rangle$, we obtained from Eq. (2)

$$\langle K_a \rangle = K_{a\infty}\left[1 - \frac{1+\bar{\omega}/6}{(1+\bar{\omega}/2)^3}\right], \tag{12}$$

where

$$\bar{\omega} = 4a(n)\varkappa\bar{\rho}/K_{a\infty}. \tag{13}$$

The formula for $\langle \bar{\mu} \rangle$ is very awkward, so that we will approximate it with the expression

$$\langle \bar{\mu} \rangle = \bar{\mu}_\infty [1 - e^{-3\bar{\rho}}(1+3\bar{\rho})]. \tag{14}$$

At low values of $\bar{\rho}$, the value of $\langle K \rangle$ computed from Eq. (11) may turn out to be smaller than the value of $\langle K_a \rangle$ calculated from Eq. (12) in view of the approximate nature of Eqs. (1) and (2). Thus, at low values of $\bar{\rho}$ it will be assumed that Eq. (11) determines the dimensionless scattering coefficient and that the attenuation coefficient is calculated by adding the scattering and absorption coefficients. This procedure is applicable in the case $K_a \ll K$.

Using Eqs. (11)-(14), we calculated the spectral and integral reflectivities of a semi-infinite layer of water droplets. The layer is being struck by hemispherical radiation, i.e., the intensity of the latter is independent of the angle of incidence. With a spherical scattering function, the solution is expressed through the moments of Ambartsumyan's function [6], which was tabulated in [7]. The results of the numerical calculations are, accurate to within several units of the third sign, approximated by the expression

$$R = \frac{1-\sqrt{\delta}}{1+\sqrt{\delta}} - 0.16\delta^{1/3}(1-\delta^{1/3}), \tag{15}$$

where $\delta = 1-\gamma$; $\gamma = (k-\alpha)/k$ — the probability of survival of a photon in a collision. Table 3 shows a comparison of the results obtained with Eq. (15) with the results of precise calculations. The transition to a nonspherical scattering function is made by means of the simi-

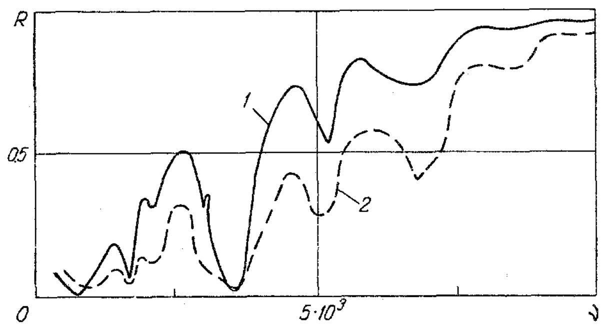

Fig. 1. Spectral reflectivity of a semi-infinite layer of water droplets with particle radii distribution (7). ν, cm^{-1}.

TABLE 3. Comparison of Precise Calculations with Calculations Using Eq. (15)

γ	R	R(15)	γ	R	R(15)
0.1	0.0217	0.0210	0.8	0.3419	0.3431
0.2	0.0462	0.0451	0.9	0.4780	0.4797
0.3	0.0745	0.0730	0.925	0.5296	0.5310
0.4	0.1074	0.1059	0.95	0.5966	0.5973
0.5	0.1466	0.1454	0.975	0.6950	0.6938
0.6	0.1947	0.1941	0.985	0.7546	0.7520
0.7	0.2565	0.2568	0.995	0.8498	0.8452

TABLE 4. Reflectivity of Layer of Water Droplets

r, µm	$\bar{R}$ $\tau_0=10$	$\bar{R}$ $\tau_0=\infty$	C, kgf/m³	r, µm	$\bar{R}$ $\tau_0=10$	$\bar{R}$ $\tau_0=\infty$	C, kgf/m³
0.33	0.68	0.69	0.069	1.65	0.68	0.68	0.009
0.65	0.71	0.73	0.019	3.35	0.62	0.62	0.021
1.0	0.70	0.71	0.012	33.5	0.21	0.21	0.28

larity relations [6]

$$1-\gamma_{sp}=\frac{1-\gamma}{1-\gamma\langle\bar{\mu}\rangle}. \tag{16}$$

Similar relations exist for optical thickness. They can be derived in a quasiuniform approximation and a first approximation by the method of spherical harmonics for a medium of arbitrary geometry. Allowing for Eq. (16), the expression for spectral reflectivity will have the form of Eq. (15), but

$$\delta=(1-\gamma)/(1-\gamma\langle\bar{\mu}\rangle). \tag{17}$$

Integral reflectivity was determined by the formula

$$\bar{R}=\frac{1}{\zeta(4)\cdot 3!}\int_0^\infty R(x)\frac{x^3}{e^x-1}\,dx, \tag{18}$$

where $x = h c\nu/kT$. Numerical integration was done by the method of trapezoids using the table of empirical results for the complex refractive index of water in [2]. In the interval of temperatures of the incident (on the layer) radiation $1000 \leqslant T \leqslant 2500$°K and mean radii $0.5 \leqslant \bar{r} \leqslant 5$ µm, the results are described with an error not exceeding 10% by the formula

$$\bar{R}=1-\exp\left\{-\frac{0.235}{\bar{r}^{0.2}}\left(\frac{T}{1000}\right)^{1.8}\right\}, \tag{19}$$

in which the mean radius of the particles is expressed in micrometers. The results of calculations of the spectral reflectivity of a layer of water droplets with size distribution (7) are shown in Fig. 1 at $\bar{r}$ = 0.5 and 5 µm. The sharply selective character of the medium is apparent from these results.

Table 4 shows the results of calculation of the reflectivity of a layer of monodispersed water droplets at optical thicknesses τ_0 = 10 and $\tau_0 = \infty$ in a first approximation by the method of spherical harmonics, as in [9], using the results of a rigorous solution conforming to Mie's theory. The temperature of the radiation incident on the layer was 1800°K. Integration over the spectrum in calculating the results in the table was done by the method of quadratures with Planck's weight function [8] with one component, so the results are an estimate. The last column in Table 4 shows values of particle concentration at which the optical thickness of the layer τ_0 = 10 at a wavelength corresponding to the component of the integration formula for a layer of thickness L = 0.2 m. The reflection coefficient in this case is only slightly lower than at $\tau_0 = \infty$, so that the value of CL from the table practically ensures the effect of a semiinfinite medium. The results of the calculations using a rigorous electromagnetic theory of light scattering by particles reveal the existence of an optimum particle size at which reflectivity will be maximal if $\varkappa \neq 0$.

NOTATION

K, dimensionless attenuation coefficient) K_a, dimensionless absorption coefficient; r, particle radius; $\bar{r}$, mean particle radius; λ, radiation wavelength; n, refractive index; $\varkappa$, absorption index; ρ, parameter for particle size; $\bar{\mu}$, mean cosine of scattering angle for minimum scattering; $\langle\bar{\mu}\rangle$, the same for minimum volume of polydispersed particles; C, particle concentration; γ, ratio of scattering coefficient to attenuation coefficient; < >, mean value; f, density of particle radii distribution; R, spectral reflectivity; $\bar{R}$, integral reflectivity; ζ, Reimann zeta function; c, speed of light; h, Planck's constant; k, Boltzmann's constant; ν, reciprocal of radiation wavelength; T, absolute temperature of radiation; L, thickness of layer; τ_0, optical thickness of layer.

LITERATURE CITED

1. G. Hulst, Light Scattering by Small Particles [Russian translation], IL, Moscow (1961).
2. V. E. Zuev, Propagation of Visible and Infrared Waves in the Atmosphere [in Russian], Sovet-skoe Radio, Moscow (1970).
3. Yu. A. Popov. "Irradiation of a layer of coarse alumina particles," Inzh.-Fiz. Zh., 31, No. 6, 996-999 (1976).
4. V. P. Pinchuk and N. P. Romanov, "Absorption cross section of spherical particles of arbitrary dimension," Zh. Prikl. Spectrosk., 27, No. 1, 109-116 (1977).
5. K. S. Shifrin and V. F. Raskin, "Spectral transmittance and the inverse problem of the theory of scattering," Opt. Spektrosk., 11, No. 2, 268-271 (1961).
6. V. V. Sobolev, Light Scattering in Planetary Atmospheres, Pergamon (1974).
7. V. V. Ivanov, Radiative Transfer and the Spectra of Celestial Bodies [in Russian], Nauka, Moscow (1969).
8. Yu. A. Popov, "Radiative transfer between the surfaces of a plane isothermal layer," Inzh.-Fiz. Zh., 15, No. 4, 734-735 (1968).
9. M. N. Otsisik, Complex Heat Exchange [Russian translation], Mir, Moscow (1976).